

# THD Minimization in Single Phase Symmetrical Cascaded Multilevel Inverter Using Programmed PWM Technique

M.Mythili, N.Kayalvizhi

**Abstract**—Harmonic minimization in multilevel inverters is a complex optimization problem that involves nonlinear transcendental equations having multiple local minima. The non linear equations are obtained from the programmed PWM technique which characterizes the low order harmonics to be eliminated. The main challenge of programmed PWM or Selective Harmonic Elimination PWM technique is to solve the equations whose solutions produce improved harmonic reduction. In this paper, optimization algorithm based on natural selection is proposed to solve the non linear equations which are more effective and time consuming than the conventional algorithm. This paper implements constrained genetic algorithm to seven-level cascaded multilevel inverter using MATLAB software and the results are presented.

**Index Terms**—Cascaded Multilevel Inverter, Genetic Algorithm, Harmonics, Optimization, Programmed PWM, Seven-level, Total Harmonic Distortion.



## 1 INTRODUCTION

GENERALLY the output voltage of the inverters must be sinusoidal. However the waveforms of practical inverters are non sinusoidal and contain certain harmonics. For low and medium power applications, square wave or quasi square wave may be acceptable but for high power applications, low distorted sinusoidal waveforms are required. By increasing number of levels in inverter the output voltage have more steps generating a staircase waveform, which has reduced harmonic distortion. There emerges need of multilevel inverter. In recent years, multilevel inverters have received more attention in industrial applications, such as motor drives, Static VAR Compensators (STATCOMs), Flexible AC Transmission System (FACTS), high voltage direct current lines, electrical drives and renewable energy systems [1].

The most attractive features of multilevel inverters are as follows [2].

- They can generate output voltages with extremely low distortion and lower  $dv/dt$ .
- They draw input current with very low distortion.
- They generate smaller Common Mode (CM) voltage, thus reducing the stress in the motor bearings. In addition, using sophisticated modulation methods, CM voltages can be eliminated.
- They can operate with a lower switching frequency.

There exist three commercial topologies of multilevel voltage source inverters: Neutral Point Clamped (NPC), Cascaded H-Bridge (CHB), and Flying Capacitors (FCs).

Cascaded multilevel inverters are based on a series connection of several single phase inverters. This structure is capable of reaching medium output voltage levels using only standard low-voltage technology components. Typically, it is necessary to connect three to ten inverters in series to reach the required output voltage. These converters also feature a high modularity degree because each inverter can be seen as a module with similar circuit topology, control structure, and modulation. Therefore, in the case of a fault in one of these modules, it is possible to replace it quickly and easily. Moreover, with an appropriated control strategy, it is possible to bypass the faulty module without stopping the load, bringing an almost continuous overall availability [3].

For improving inverter performance and output quality, different methods have been suggested. The first of them is using various switching strategies, such as Sinusoidal Pulse Width Modulation (SPWM), Selective Harmonic Elimination PWM (SHEPWM) or Programmed PWM, Space Vector Modulation (SVM), Optimized Harmonic Stepped Waveform (OHSW) and Optimal Minimization of Total Harmonic Distortion (OMTHD).

In this, Selective Harmonic Elimination PWM has been a research topic since the early 1960's, first examined in and developed into a mature form during the 1970's. SHE offers several advantages compared to traditional modulation methods including acceptable performance with low switching frequency to fundamental frequency ratios, direct control over output waveform harmonics, and the ability to leave triplen harmonics uncontrolled to take advantage of circuit topology in three phase systems. These key advantages make SHE a viable alternative to other methods of modulation in applications such as ground power units, variable speed drives, or dual-frequency induction heating [4].

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In SHEPWM method, the objective is elimination of low order harmonics, while the fundamental harmonic is satisfied. If this goal cannot be obtained, the highest possible harmonics optimization is desired. In this approach, by solving  $S$  equations,  $(S-1)$  low order harmonics from the fifth order can be eliminated and the fundamental component is satisfied. Solving SHEPWM nonlinear equations is a major problem in obtaining switching angles. Several methods have been suggested for solving the equations such as Newton Raphson (NR) method [5], theory of resultant [6] which are based on numerical iterative techniques and also some optimization algorithms such as Genetic Algorithm (GA) [7], Ant Colony Optimization (ACO) [8] and Bee Algorithm (BA) [9] which are simpler than iterative techniques and can be used for any number of levels.

In this paper, Genetic algorithm is applied to minimize the low order harmonics, as well as to satisfy the desired fundamental component. The algorithm is proposed here is used to find the optimized switching angles than the conventional one and thus reduced THD is obtained.

## 2 CASCADED MULTILEVEL INVERTER

A cascaded multilevel inverter consists of a series of single phase full bridge inverter units. The general function of this multilevel inverter is to synthesize a desired voltage from several separate DC sources, which may be obtained from batteries, fuel cells or solar cells. Each separate DC source is connected to a full bridge inverter. The cascaded multilevel inverter does not require any voltage clamping diodes or voltage balancing capacitors like other two topologies.

### 2.1 Seven-Level Cascaded Multilevel Inverter

The seven-level multilevel inverter is obtained by cascading three full bridge inverter circuits. The three full bridge inverters are connected in series and a single phase output is taken. Each full bridge is fed from separate DC source. The number of output levels  $m$  in each phase is related to number of full bridge inverter units  $n$  by,

$$m=2n+1$$

Here number of levels is seven, hence number of inverter circuits connected in series is three. The single phase seven-level topology of cascaded H-bridge multilevel inverter is shown in Fig.1.

Each H-bridge is fed with the same value of DC voltage hence it can be called as symmetrical cascaded multilevel inverter. Each full bridge inverter can generate three different voltage outputs:  $+V_{dc}$ , 0, and  $-V_{dc}$ . The phase output voltage is synthesized by sum of three inverter outputs,

$$V_{an} = V_{a1} + V_{a2} + V_{a3}$$

The seven-level output waveform is obtained by different switching combinations.

The switching angles obtained using genetic algorithm

are three angles. These three angles are used for giving pulses to twelve switches. The switching pattern for single phase seven-level topology of cascaded H-bridge multilevel inverter is shown in Table 1.

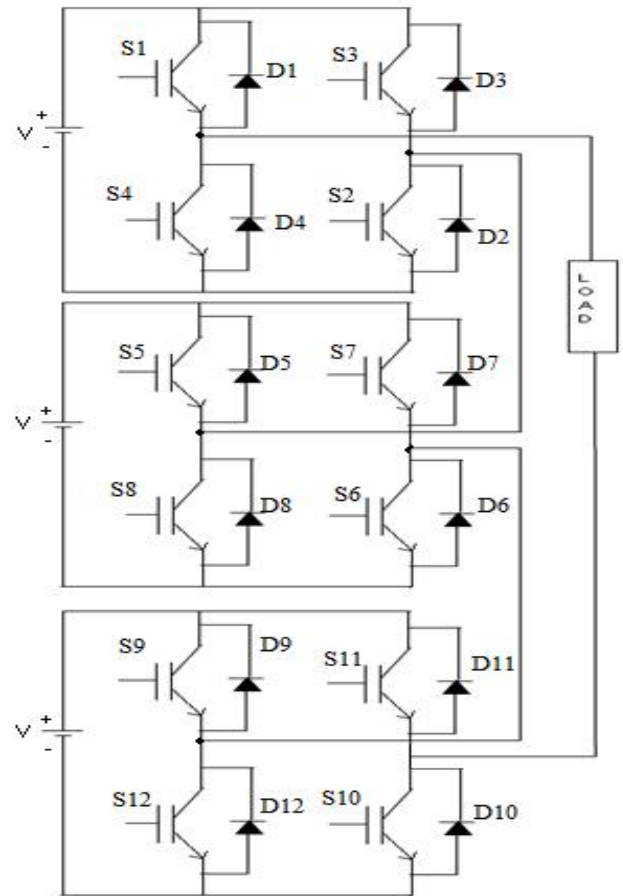


Fig.1 Single Phase 7-Level Topology of Cascaded H-Bridge Multilevel Inverter

TABLE 1

SWITCHING PATTERN FOR SINGLE PHASE SEVEN-LEVEL TOPOLOGY OF CASCADED INVERTER

VOLTAGE	SWITCHING STATES											
	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12
+V	1	1	0	0	0	1	0	1	0	1	0	1
+2V	1	1	0	0	1	1	0	0	0	1	0	1
+3V	1	1	0	0	1	1	0	0	1	1	0	0
0	0	1	0	1	0	1	0	1	0	1	0	1
-V	0	0	1	1	0	1	0	1	0	1	0	1
-2V	0	0	1	1	0	0	1	1	0	1	0	1
-3V	0	0	1	1	0	0	1	1	0	0	1	1

By using the above switching patterns seven-level output can be obtained. The output voltage waveform of seven-level inverter is shown in Fig. 2. In the figure  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  represents the optimized switching angles which are used for harmonic reduction.

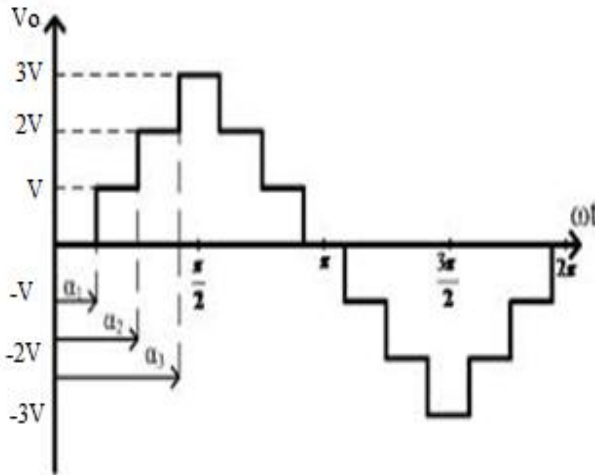


Fig.2. Output Voltage Waveform of 7-Level Cascaded Multilevel Inverter

### 3 SELECTIVE HARMONIC ELIMINATION PULSE WIDTH MODULATION (SHEPWM)

The Selective Harmonic Elimination PWM or Programmed PWM technique is based on fundamental frequency switching theory and dependent on the elimination of defined harmonic content orders. The main idea of this method is based on defining the switching angles of harmonic orders to eliminate and obtaining the Fourier series expansion of output voltage. This allows lower switching frequencies to be used which led to lower losses and higher efficiency. In general Fourier series is given by,

$$V(\omega t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \quad (1)$$

In this case Fourier series expansion of output voltage waveform is given by,

$$V(\omega t) = \sum_{n=1}^{\infty} (b_n \sin n\omega t) \quad (2)$$

Where

$$a_0 = a_n = 0 \text{ (due to quarter wave symmetry)}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} V_{dc} \sin n\omega t d\omega t \quad (3)$$

From Fig.2, for quasi square wave equation (3) becomes

$$b_n = \frac{2}{\pi} \int_{\alpha}^{\pi-\alpha} V_{dc} \sin n\omega t d\omega t \quad (4)$$

On solving equation (4) we get,

$$b_n = \frac{4V_{dc}}{n\pi} \cos n\alpha \quad (5)$$

For 7-level cascaded multilevel inverter for three dc sources it is given as,

$$b_n = \frac{4V_{dc}}{n\pi} \sum_{i=1}^s \cos n\alpha_i \quad (6)$$

Where  $n=1, 5, 7$  and  $s=3$  which represents number of DC sources. The objective of SHEPWM is to eliminate lower order harmonics while remaining harmonics can be removed with filter. In this number of harmonics that can be eliminated is equal to  $s-1$  i.e., 2 so fifth and seventh harmonics are taken. So, to satisfy the fundamental harmonic component and eliminate the fifth and seventh harmonics, three nonlinear equations with three angles are provided in,

$$b_1 = V_1 = \frac{4V_{dc}}{\pi} [\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3)] \quad (7)$$

$$b_5 = V_5 = \frac{4V_{dc}}{5\pi} [\cos(5\alpha_1) + \cos(5\alpha_2) + \cos(5\alpha_3)] \quad (8)$$

$$b_7 = V_7 = \frac{4V_{dc}}{7\pi} [\cos(7\alpha_1) + \cos(7\alpha_2) + \cos(7\alpha_3)] \quad (9)$$

To eliminate fifth and seventh harmonic  $V_5$  and  $V_7$  are set to zero in the equation (8) and (9). To determine the switching angles the following equations must be solved,

$$\cos(\alpha_1) + \cos(\alpha_2) + \cos(\alpha_3) = 3M \quad (10)$$

$$\cos(5\alpha_1) + \cos(5\alpha_2) + \cos(5\alpha_3) = 0 \quad (11)$$

$$\cos(7\alpha_1) + \cos(7\alpha_2) + \cos(7\alpha_3) = 0 \quad (12)$$

Here  $M$  represents modulation index varies from 0 to 1.  $M$  is given by,

$$M = \frac{\pi V_1}{12V_{dc}} \quad (13)$$

The switching angles  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  must be less than  $\pi/2$ . The equations are solved by Newton Raphson (NR) method and resultant theory in the literature. But it is time consuming and needs initial guess for solving the equations. Hence evolutionary algorithms are used for solving this type of non linear equations. Here genetic algorithm is proposed to solve these equations.

#### 3.1 Proposed Genetic Algorithm

Genetic Algorithm is a method used for solving both constrained and unconstrained optimization problems based on natural selection. It imitates biological evolution by using genetic operators referred to as reproduction, crossover, mutation etc. The genetic algorithm is simple and applicable to problems with any number of levels, without the extensive derivation of analytical expressions, for both eliminating and minimizing harmonics. This algorithm is used for optimizing switching angles.

The structure of a simple GA consists mainly of three operators. A selection operator, a crossover operator which acts on a population of strings to perform the required reproduction and recombination, and a mutation operator which randomly alters character values, usually with a very low probability. GA technique is used for its ability to deal with complicated problems where analytical formula is not yet possible. In the multi-objective SHE problem, if there is no set of angles that will satisfy the SHE

equation, the analytical approach will not return an answer. The GA, on the other hand, will always return an answer that will not exactly solve all equations but instead will give answers that are very close to the solutions. Thus instead of eliminating harmonics it minimizes them.

The steps followed in the proposed genetic algorithm are as follows:

1. select the population type
2. Initialize the population
3. Evaluate the fitness function of each individual
4. Minimize the fitness function that satisfies the constraints (10)-(12).
5. Pick the best individuals
6. Create a new offspring using crossover and mutation operations
7. If number of iterations is less than 100, repeat the process otherwise terminate the process.

#### 4 IMPLEMENTATION OF GENETIC ALGORITHM FOR OBTAINING SWITCHING ANGLES

The SHEPWM equations involve trigonometric terms which are difficult to solve. The methods like Newton Raphson method and resultant theory are normally used to solve the equations. But when number of levels increases it becomes complex to use these equations. Hence optimization algorithms are used to solve these types of problems. Here the algorithm based on genetic algorithm is implemented to solve these types of equations. The switching angles are determined using genetic algorithm by optimizing the fitness functions and satisfying the constraints. The steps for formulating a problem and applying GA are as follows:

1. Select binary or floating point strings.
2. Find the number of variables specific to the problem; this number will be the number of genes in a chromosome. In this application the number of variables is the number of controllable switching angles which is the number of H-bridges in a cascaded multilevel inverter. A seven-level inverter requires three H-bridges; thus, each chromosome for this application will have three switching angles.
3. Set a population size and initialize the population. The population used here is 20 chromosomes, each containing three switching angles. The population is initialized with random angles between 0 degree and 90 degree taking into consideration the quarter wave symmetry of the output voltage waveform.
4. The most important item for the GA to evaluate the fitness of each chromosome is the objective function. The objective of this study is to minimize specified harmonics; therefore the objective function has to be related to these harmonics. The harmonics taken here are fifth and seventh harmonic hence objective function is,

$$objfun = \frac{\sqrt{\sum_{n=5,7} \left( \frac{1}{n} \sum_{k=1}^3 \cos(n\alpha_k) \right)^2}}{\sum_{k=1}^3 \cos(\alpha_k)} \quad (14)$$

5. The Fitness Value (FV) is given by,

$$FV = \frac{1}{1+objfu_n} \quad (15)$$

The switching angle set producing the minimum FV is the best solution of the iterations.

6. The GA is usually set to run for a certain number of iterations (100 in this case) to find an answer. After the first iteration, FVs are used to determine new offspring. These go through crossover and mutation operations and a new population is created which goes through the same cycle starting from FV evaluation until the solution is found that satisfy the constraints (10)-(12).

##### 4.1 Switching Angles Obtained Using Proposed Genetic Algorithm

The Proposed Genetic Algorithm coding is written using MATLAB m-file. The objective function is written as user defined function in a separate file. When the genetic algorithm coding is running it calls the objective function while evaluating fitness function. The solution is obtained when the fitness value is minimum and constraints are satisfied. The switching angles are obtained for various modulation indexes from 0.4 to 0.8. The switching angles obtained for various modulation indexes are shown in Fig. 3.

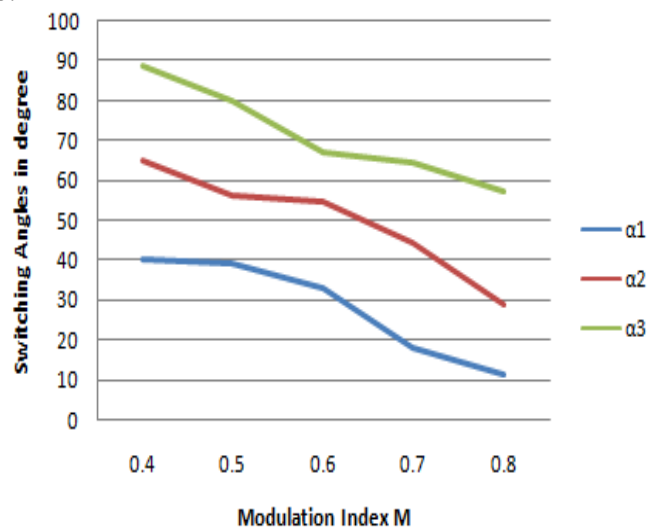


Fig.3. Switching Angles for Various Modulation Index Obtained by GA

#### 5 SIMULATION RESULTS

The simulation model of single phase seven level inverter is shown in Fig.4. It consists of three full bridge inverters connected in series. The switching pulses are given from the switching circuit to the IGBT switches. The single phase AC output is given to the load. The input voltages for all

the three full bridge inverters are same.

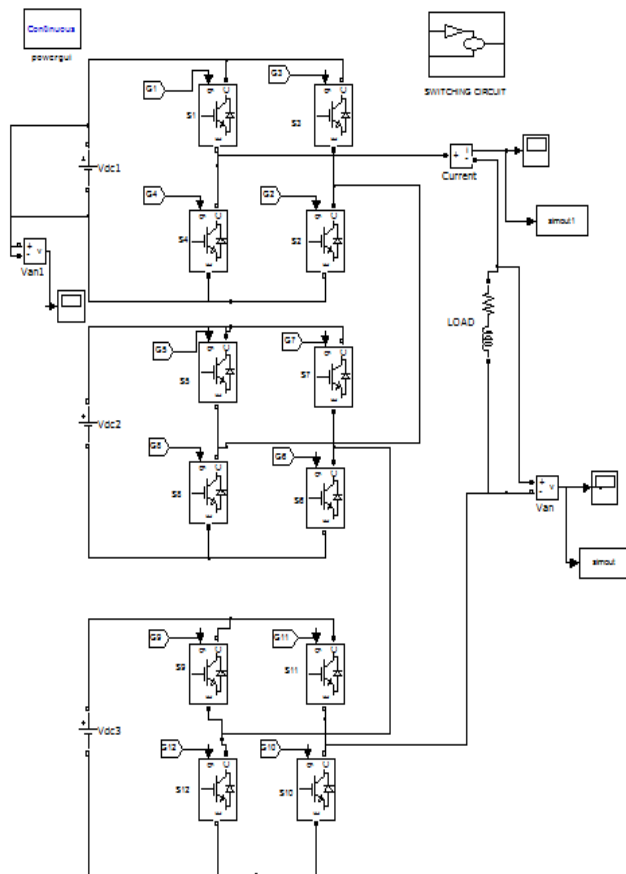


Fig.4.Simulation Model of Single Phase Seven-Level Cascaded Multilevel Inverter

The THD values for different values of modulation index are shown in the Fig.5. From the Figure, it can be inferred that when modulation index is higher, the THD value is lower. Hence the simulation results are shown for  $M=0.8$ .

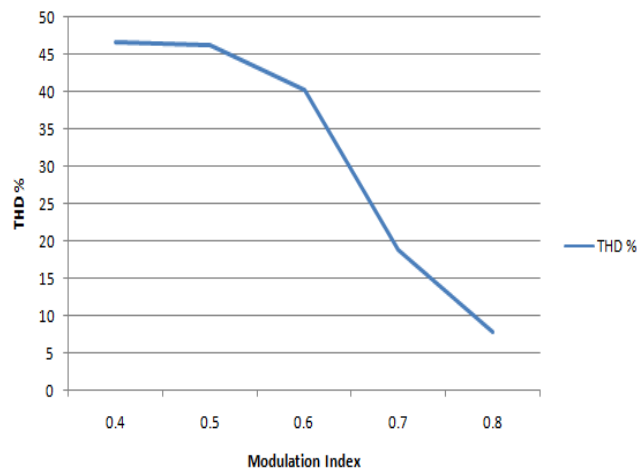


Fig.5. THD for Various Values of Modulation Index

### 5.1 Simulation Results for $M=0.8$

The input voltage given for multilevel inverter is 100V. The switching angles obtained using GA are given to the switching circuit whose output is given to switches of cascaded multilevel inverter. The switching Pulses are shown in Fig.6.

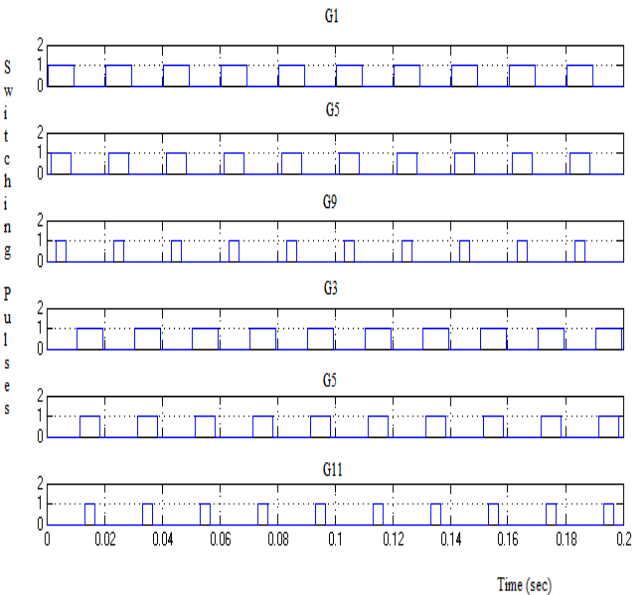


Fig.6. Switching Pulses for Seven-Level Cascaded Multilevel Inverter

The output voltage of the seven-level cascaded multilevel inverter is shown in Fig.7. The output voltage is 300V which is thrice the input voltage. The output frequency is 50Hz. The seven levels of the output are obtained.

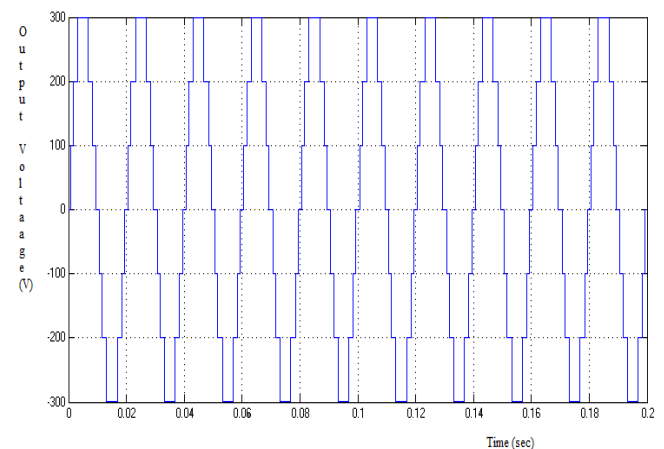


Fig. 7. Output Voltage Waveform for Seven-Level Cascaded Multilevel Inverter

The FFT analysis of the output voltage waveform is done to estimate THD. The FFT analysis of the output voltage waveform is shown in Fig.8. The THD obtained for seven-level multilevel inverter is 7.72%. The fifth and



seventh order component also minimized. The desired fundamental component is 305.57 and the obtained value is 303 thus it satisfies the fundamental component.

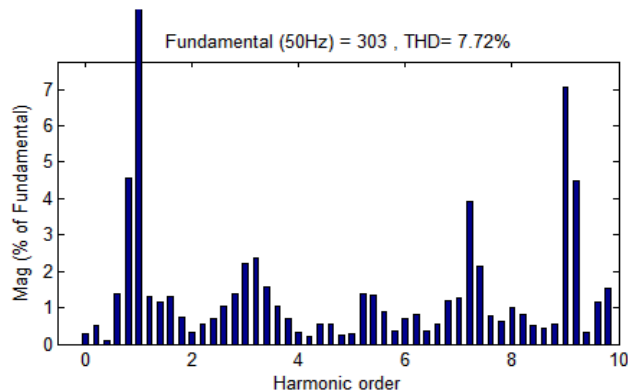


Fig.8.FFT Analysis of Output Voltage waveform of Cascaded Multilevel Inverter

## 6 CONCLUSION

In this paper a method based on genetic algorithm is used to solve the non linear transcendental equations. These equations determine the switching angles which are used to minimize the THD. The effectiveness of the applied method is verified using the simulation results. This work can be extended to multilevel inverters with reduced number of switches for further enhancement of output waveform.

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